Brooke Hassig

Mrs. Tallman

Advanced Placement Calculus

9 March 2015

The Derivative vs. The Integral

 Calculus is filled with hundreds of concepts that intrigue students on a daily basis. Two closely related topics that really spark interest in the mind of not only students, but everyone around the world are derivatives and integrals. The derivative is the instantaneous rate of change of the dependent variable with respect to the independent variable. The notation of the derivative is f’(x). The integral of the derivative can either be indefinite of definite. With an indefinite integral, the antiderivative of the function is taken, and then C, a constant, is added to the antiderivative since it is not evaluated at endpoints. The notation for the integral is shown below.

With a definite integral, the antiderivative of the function is taken, and then that is evaluated at the endpoints, b and a. The function with the endpoint a plugged in is subtracted from the function with the endpoint b plugged in.



Figure 1. Derivative and Integral Graph (“MatematicasVisuales: Cubic”)

 The original function in Figure 1 is the cubic function which is also the graph of displacement. The derivative of the function quadratic and is also known as the graph of velocity. The integral of the graph is the area between the two functions which is the area under the derivative evaluated at the end points, in this situation, the intersection points of both graphs. So given the graph of displacement, the integral of the graph would be the area between the functions, and the derivative would be the graph of velocity. When taking the derivative of the displacement function, the cubed goes away thus making the velocity function just the original unit squared.



Figure 2. Function and Derivative Graph (“MatematicasVisuales: Quadratic”)

 The original function in Figure 2 is the quadratic, which is also the graph of velocity. The derivative of the function is linear and is also known as the graph of acceleration. The integral of the graph is the area between the two functions which is the area under the derivative evaluated at the end points, in this situation, since there are two endpoints, the definite integral would have to be taken. So given the graph of velocity, the definite integral of the graph would be the area between the functions, the displacement, and the derivative would be the graph of acceleration. Velocity is over time, thus making the units for the graph in units/time2. When taking the derivative of the velocity function, the squared goes away thus making the acceleration function just a number. The integral of the graph would be the units to displacement. For

 When the sign of the first derivative is positive, it means the function has an increasing velocity. When the sign of the first derivative is negative, it means the function has a decreasing velocity. If the sign of velocity and acceleration, the first and second derivative, are the same, it means that the function is speeding up. If the signs are different between the derivatives, it means the function is slowing down. The sign for the second derivative determines the direction. If the sign is negative, the function is moving in the negative direction and if the sign is positive it is moving in the positive direction.

![[image]]()

Figure 3. Problem 1 Graph

The critical points of the graph in Figure 3 are -5,-3, 1, and 4. These points are considered critical points because they are zero on the first derivative graph. These points represent either maximum or minimum points in the original graph. The concavity of the graph found through the second derivative determines if the points are maximums or minimums. When the slope of the first derivative is positive, the original function is concave up. If the slope of the first derivative is negative, the original function is concave down through those values. In the graph of Figure 3, since the graph is increasing from x-values of -5 to -4, -1 to 1, and 1 to 2 the graph is concave up in those regions. From x-values of -4 to -1 and 2 to 4, the graph is decreasing so the values are concave down in those regions. When the graph is concave up, this is where a minimum occurs and when the graph is concave down that is where a maximum occurs. This is called the second derivative test. When f’(x) is equal to zero, that is where the minimums and maximums of the original graph are. The minimums in the case of problem 1 are x=-5 and x=1. The absolute minimum of the graph is (1,3) because the point is a zero on the derivative graph, it switched from positive to negative slope, and because the only other consideration would be (-5,0) and since that is an endpoint, we cannot be for sure that it is the absolute minimum. The maximums in problem 1 are (-3,0) and (4,0). When the slope of the graph changes, that is where a point of inflection occurs. In the case of problem 1, the points of inflection are (-4,0); (-1,0); and (2,0) since the slope is changing.

When given the integral and asked to find the original value, first derivative, and second derivative of a point, the process is very simple. For example, with the integral

we are asked to find g(3), g’(3), and g’’(3). To find g(3) the first Fundamental Theorem of Calculus will be used. The sample equation is shown below.

When finding g(3), the answer would be 2.5. The integral has to be split up because you are evaluating two different parts of the original function. To find g’(3) the Second Fundamental Theorem of Calculus will be used. The second Fundamental Theorem states to find the integral, you multiply the upper limit by f’(x). A sample calculation is shown below.

On the graph in Figure 3, f’(3) is 1. The derivative of x is just 1, which is how you get g’(3) equal to 1. When 3 is plugged in to f’(3), you get g’(3) to equal 1. The second derivative is the slope of that line. The slope of the line at f’(3) is negative, which would make g’’(3) equal to -1.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X | F(x) | F’(x) | G(x) | G’(x) |
| 1 | 3 | 4 | 2 | 5 |
| 2 | 9 | 2 | 3 | 1 |
| 3 | 10 | -4 | 4 | 2 |
| 4 | -1 | 3 | 6 | 7 |

Figure 4. Problem 2

The first Fundamental Theorem of Calculus states that if *f* is an integrable function and if , then . Essentially what is happening is you are taking the antiderivative of *f(x),* in this case the antiderivative would be *g(x)*, and evaluating at the endpoints. The second Fundamental Theorem of Calculus states if where a is constant, and *f* is continuous in the neighborhood of a, then . Basically, you evaluate the integral at the top value, the limit, and multiply by the derivative of the top value. In relation to Figure 4, the answer to the integral of

would be found through the second Fundamental Theorem of Calculus. When finding w’(3), the answer would be -2. The answer is -2 because w’(x)=f (g(x))\* g’(x). The Fundamental Theorem states you multiply f(t) by the derivative of the upper limit. The derivative of the upper limit is g’(x). The number 3 is then plugged in for each of the x values. G(3) is 4 and f(4) is -1 as seen in Figure 4. G’(3) is 2 according to Figure 4, so when those values are multiplied together you get the answer of w’(3) to be -2.

The Mean Value Theorem states if *f* is differentiable for all x in (a,b) and *f* is continuous for all x in [a,b] then there exists at least one number x=c in (a,b) such that

F’(c) is the instantaneous rate of change at x=c. The instantaneous rate of velocity is equal to the average velocity. The function evaluated and subtracted at the endpoints over the endpoints subtracted from each other is equal to the slope of the secant line from a to b. For the second part of problem 2, there must be a value c for 1 < c < 3 such that H’(c) = -5, because the function is continuous and differentiable. Based on Figure 4 and the statement that H(c)=f(g(c))-6, H(3) is equal to -7 and H(1) is equal to -3. When those are subtracted from each other, the answer is -10. b-a, or 3-1, is equal to 2. When -10 is divided by 2, the answer is -5, therefore H’(c) is -5 when the interval is plugged into the equation above.

 The Intermediate Value Theorem states if *f* is continuous for all x in [a,b] and f(a)<y<f(b) then there is a number c in (a,b) such that f(c)=y. You essentially plug a and b into the original function to get the range and then conclude if that the given f(c) value is in the interval. If c is not in the interval, the Intermediate Value Theorem cannot be applied. In part a of problem 2, point r has to be in between 1 and 3. H(r)=f(g(r))-6 and at point 1, H(r) is equal to 3. At point 3, H(r) is equal to -7. -5 has to be in this integral, because you cannot go from 3 to -7 without hitting -5 on a continuous function. Therefore, by the Intermediate Value Theorem there exists some r in (0,3) such that H(r)=-5.

 In problem 2, part d, G-1 is the inverse function of G and a line tangent to the graph k(x)= G-1(x) at x=2 has to be written. A sample calculation is shown below.

Like other inverse functions, the derivative of the inverse function times the derivative of the initial function equals 1. With that being said, the inverse is equal to 1 over the derivative of the initial function. Evaluating the function at 2 found the tangent line of the function. Since g’(x) is an inverse the inverse value at x=2 will be used. The inverse value is then 1 which is why g’(x) was evaluated at 1 in the calculation above. The derivative found is the slope which is then plugged in to point slope form using the values (2,1).

 In problem 2, part e, H(x)=xB(x) where B(x) is equal to the inverse of F(x). H(x) is then equal to x multiplied by the inverse of F(x). To find H’(x) the product rule will be utilized. The H’(x) equation would be:

When 3 is plugged in it would look like the equation show below:

F(3) as shown in Figure 4, is equal to 10. F’(3) is -4 according to the chart in Figure 4. When everything is multiplied and added out, the complete answer to H’(3) is equal to 1.75.

 When the endpoints of an integral are swapped, the integrals are opposite of each other. The integral shown below is evaluated from b to a.

The next integral is evaluated from a to b. This change or swap in endpoints will result in the solution of the integrals sign to be opposite from the previously shown integral. For example, if the integral has a positive solution then the integral has a negative solution.

The solution of the integrals remain the same, the sign is just swapped meaning that the area is changing from either positive to negative or negative to positive.

 In conclusion, math is a very powerful topic. Calculus is filled with hundreds of concepts that challenge the mind daily. Two calculus topics that really challenge the mind of not only students, but everyone around the world are derivatives and integrals. Those topics can be used to solve many different things as discussed above. Although derivatives and integrals look difficult, they are actually not that hard to understand. The practice of problems always helps to make them easier and to improve your consistency of getting the answers correct. Mastering the concepts of derivatives and integrals is another step to understanding the hundreds of other calculus concepts that students are eager to learn on a daily basis.

Works Cited

"MatematicasVisuales | Polynomial Functions and Derivative (2): Quadratic Functions." *MatematicasVisuales | Polynomial Functions and Derivative (2): Quadratic Functions*. N.p., n.d. Web. 07 Mar. 2015. <http://www.matematicasvisuales.com/english/html/analysis/derivative/quadratic. html>.

"MatematicasVisuales | Polynomial Functions and Derivative: Cubic Functions." *MatematicasVisuales | Polynomial Functions and Derivative: Cubic Functions*. N.p., n.d. Web. 07 Mar. 2015. <http://www.matematicasvisuales.com/english/html/analysis/derivative/cubic.html >.