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Advanced Placement Calculus

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The Revolution of Solids

Calculus is filled with hundreds of topics that can be used in everyday life. One calculus topic that specifically relates to this idea is solids of revolution. Have you ever wondered what the volume of your donut was? Or of a block of cheese? Or even a hershey kiss? Solids of revolution is a calculus topic that can help answer these extraordinary questions in a simple way. The topic of solids of revolution can be mastered through the learning of several steps. First, finding the area under a curve or between curves is the beginning step of understanding this topic. The next step is actually finding the volume of the solid through rotation around an axis or a given line with the shell, disk, or ring methods. Finally, the last step is finding the volume of an arbitrary object while utilizing the slab method is the final idea that concludes the topic of solids of revolution.

Finding the area between curves or under a curve is a simple process. The area between the curves can essentially be thought of as the space bound between two functions. The area under the curve is the space bound between a function and the given axis. To find this area, the definite integral is utilized. The proper notation for the

definite integral of area under a curve is:

The definite integeral above allows students to find the area under the curve through calculus. The points of b and a represent the end and beginning points on either the x or y axis of where the area is calculated between. These end points enclose the area between the function and the axis. F(x) is the given function in the integral and dx relates to how the graph is being sliced in order to find the area. If you wanted to find the area using x-points, then the space would be cut vertically also known as a dx cut. If you wanted to find the area using y-points, then the space would be cut horizontally also known as a dy cut. With whatever cut you decide to use, the function has to be in corresponding terms of x or y to find the correct area. All of the space enclosed by the function, the two end points, and the axis is the area under the curve from point a to point b otherwise known as the integral shown above. The integral is then evaluated through the Fundamental Theorem of Calculus. The Fundamental Theorem involves taking the antiderivative of the function and then plugging in the end points of b and a. After the end points are plugged in to the antiderived equation, the antiderivative evaluated at point a would be subtracted from the antiderivative evaluated at point b. The answer to this would be the area in the given units squared.

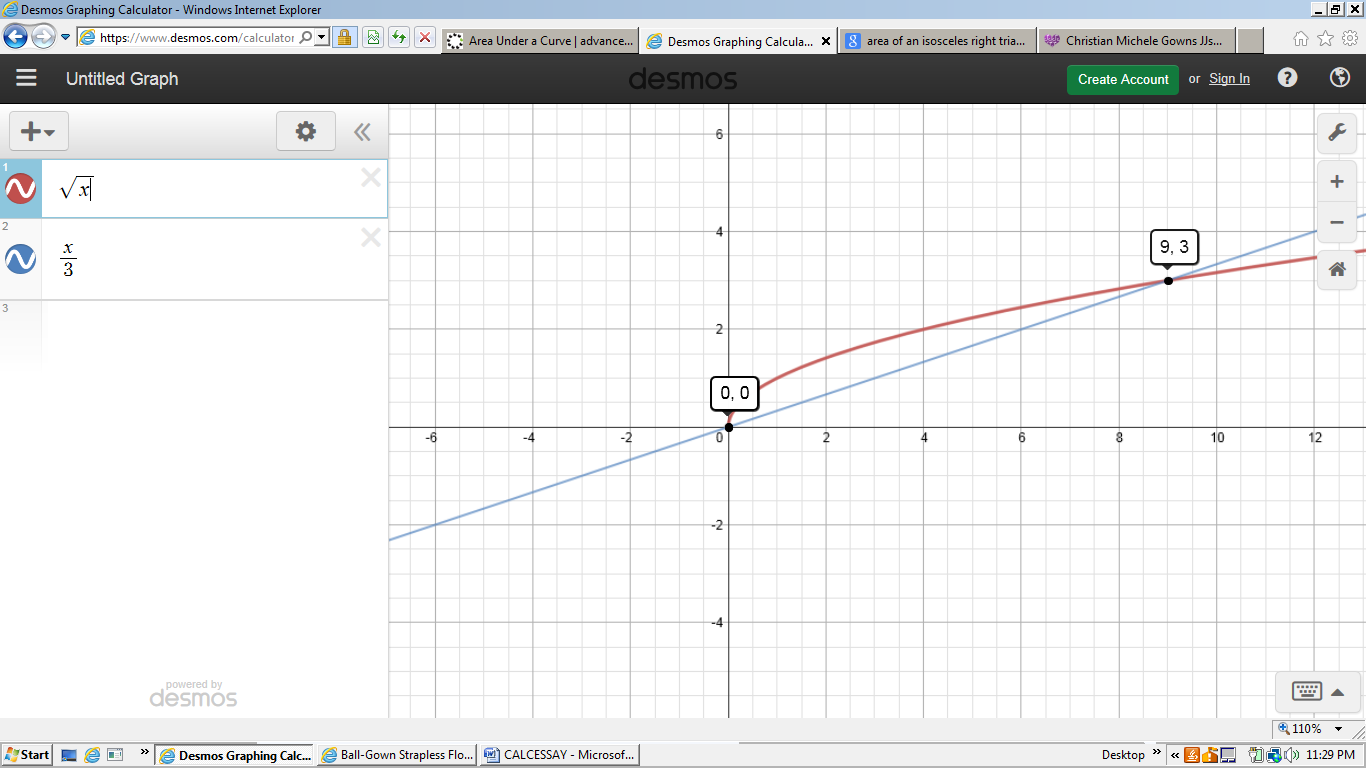


Figure 1. Area between the Curves (“Desmos Graphing Calculator”)

In the given example problem, and were graphed in order to find the area between the curves. The functions are graphed above in Figure 1. To find the area between these two curves, a different definite integral would be used. The definite integral for this situation is:

The definite integral above allows students to find the area between the two given functions through calculus. The points of b and a represent the end and beginning points on either the x or y axis of where the area is calculated between. In the situation of Figure 1, the end points are based on the x axis. Point a is 0 since it is the starting point and point b is 9 since it is the end point. These end points enclose the area between the two functions. F(x) is the top function or in this situation and g(x) is the bottom function or. To determine which function is f(x) and which is g(x), it always goes top minus the bottom for dx cuts or right minus left for dy cuts. Basically, the top or most right function is f(x) and the bottom or most left function is g(x). The dx relates to how the space between the curves is being sliced in order to find the area. If you wanted to find the area using x-points, then the space between the curves would be cut vertically also known as a dx cut. If you wanted to find the area using y-points, then the space would be cut horizontally also known as a dy cut. With whatever cut you decide to use, the function has to be in corresponding terms of x or y to find the correct area. Since the endpoints stated above are x points, the graph is being sliced with a dx or vertical cut. All of the space enclosed by the two functions and the two end points is the area between the curves from point a to point b otherwise known as the integral displayed above. A sample set up and solution is displayed below to help with the understanding of the concept.

As previously stated, since the graphs are bound together, the intersection points then become the end points for the area. The difference between and can be evaluated using integrals or the Fundamental Theorem of Calculus at the end points in order to find the area. The thicknesses of the disks are dx, because the graph is being sliced perpendicular to the x-axis. The area between the two curves in Figure 1 is 4.5 units².

The next step in mastering the concept of solids of revolution is understanding the different types of methods for finding volume. The three different methods for finding volume are the disk method, the ring method, and the shell method. The disk method is used when one function is rotated around an axis or given line. The ring method is used when two functions are rotated around an axis or given line. The shell method is used when one or two functions are rotated around an axis or given line, but using the opposite cut. For example, while using the shell method to rotate a function around the y-axis the cut would be a dx cut and the points would be x-points even though it is rotated around the y-axis.

When a function is rotated around the x-axis, the cut is vertical also known as a dx cut. When a function is rotated around the y-axis, the cut is horizontal also known as a dy cut. The cut of the function helps find the volume because it resembles the thickness of the graph.

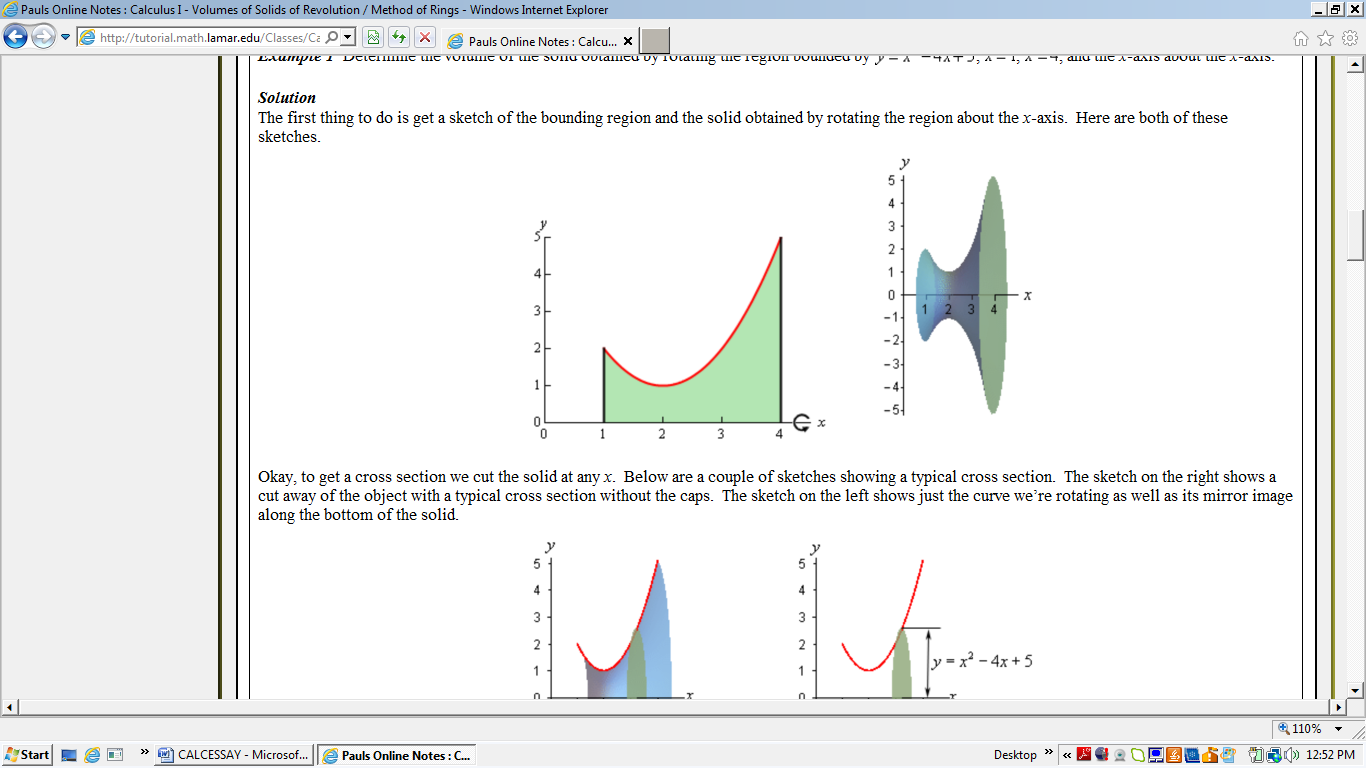
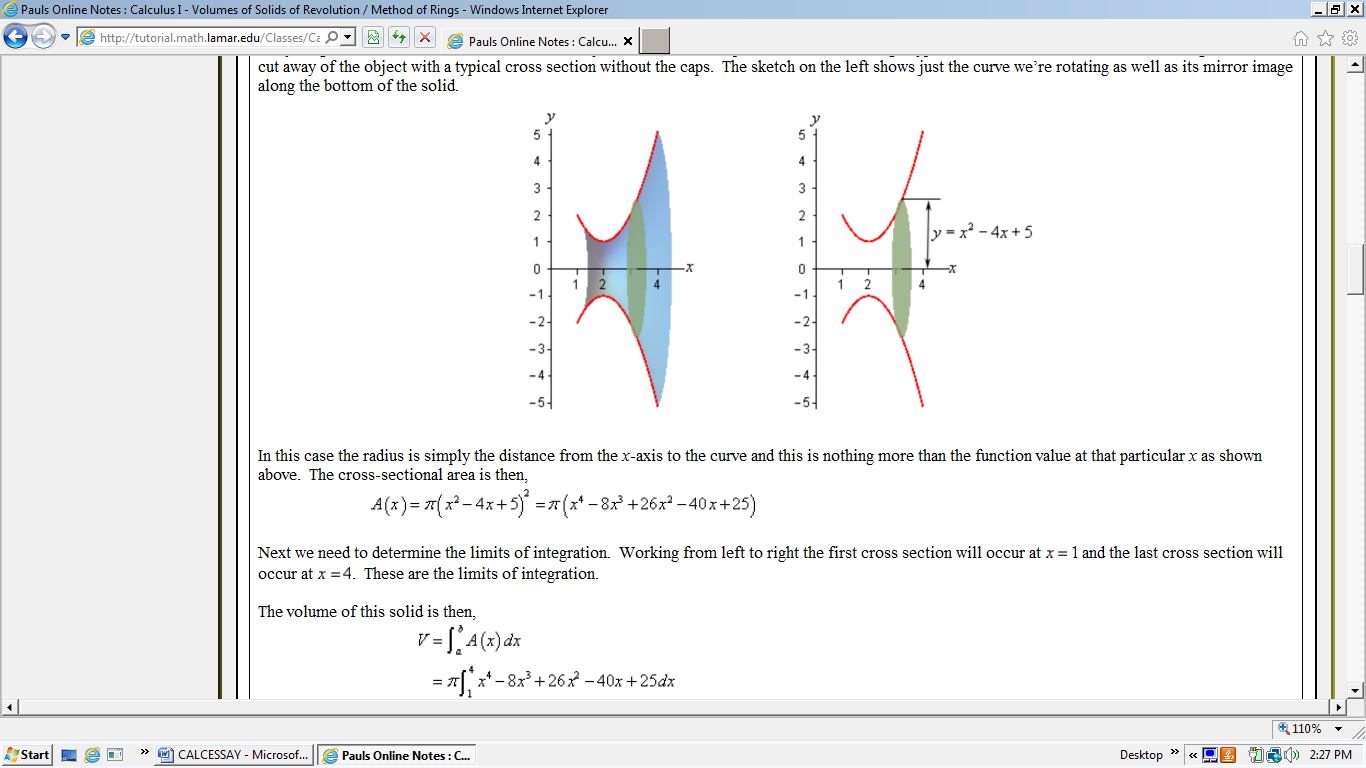
 

Figure 2. The Disk Method (“Pauls Online Notes: Rings”)

The green shaded area above in Figure 2 is bound between points 1 and 4, the function itself, and the x-axis. The green shaded area is then rotated around the x-axis. The shaded rotated region is cut into an infinite amount of slices essentially making disks when rotated around the x-axis. To find the volume, the definite integral for disks is utilized. The definite integral is:

In the definite integral above, the points b and a are the endpoints of the rotated disk method. In relation to Figure 2, point b would be 4 and point a would be 1. The radius is the distance from the axis or line to the function itself, essentially half of the width of the disk created. In relation to Figure 2, the radius of the graph is the distance from the x-axis to the function itself. The radius in this case would just be the function itself. Since the graph is rotated around the x-axis, the graph would be sliced into a dx cut thus utilizing x coordinates as end points. The entire integral is then multiplied by because the area of a disk is and when multiplied by dx, the thickness, you get the volume of the infinite amount of disks added all together which creates the rotated solid.

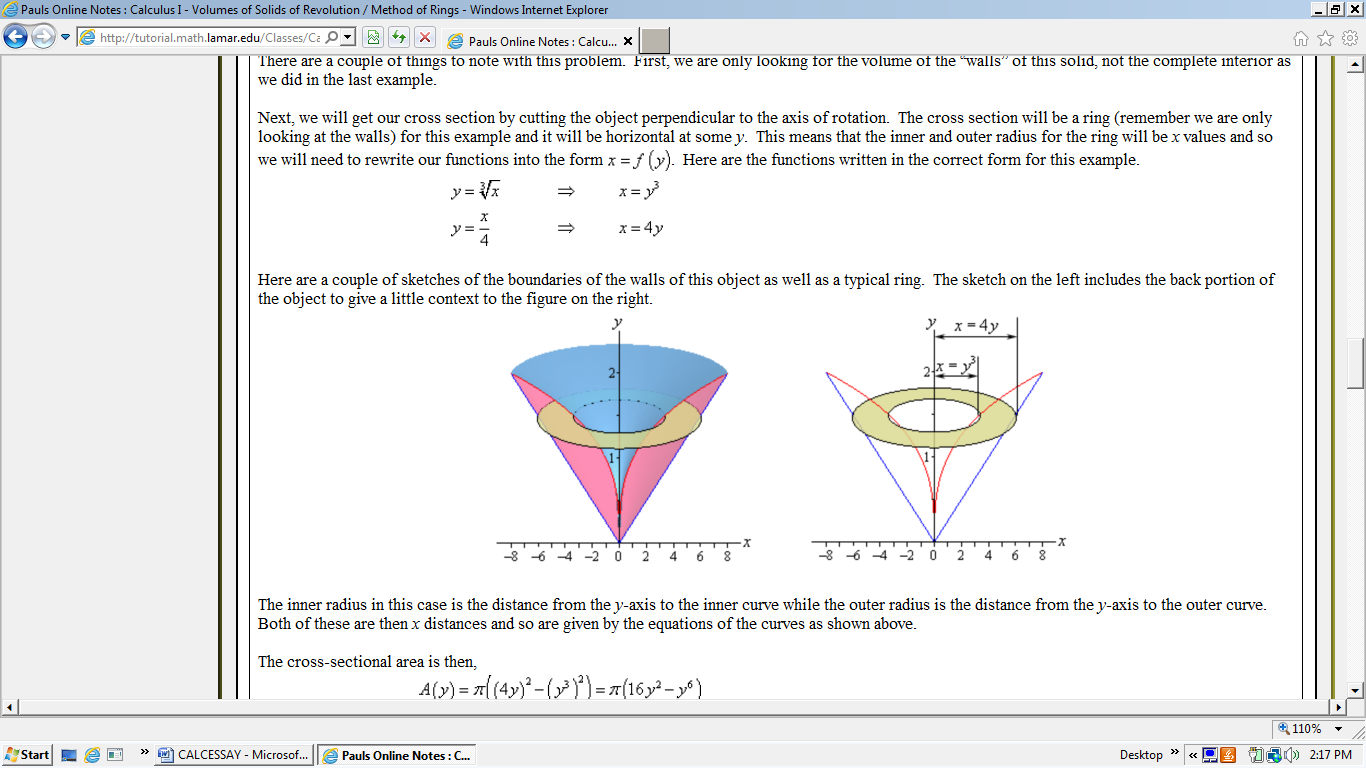
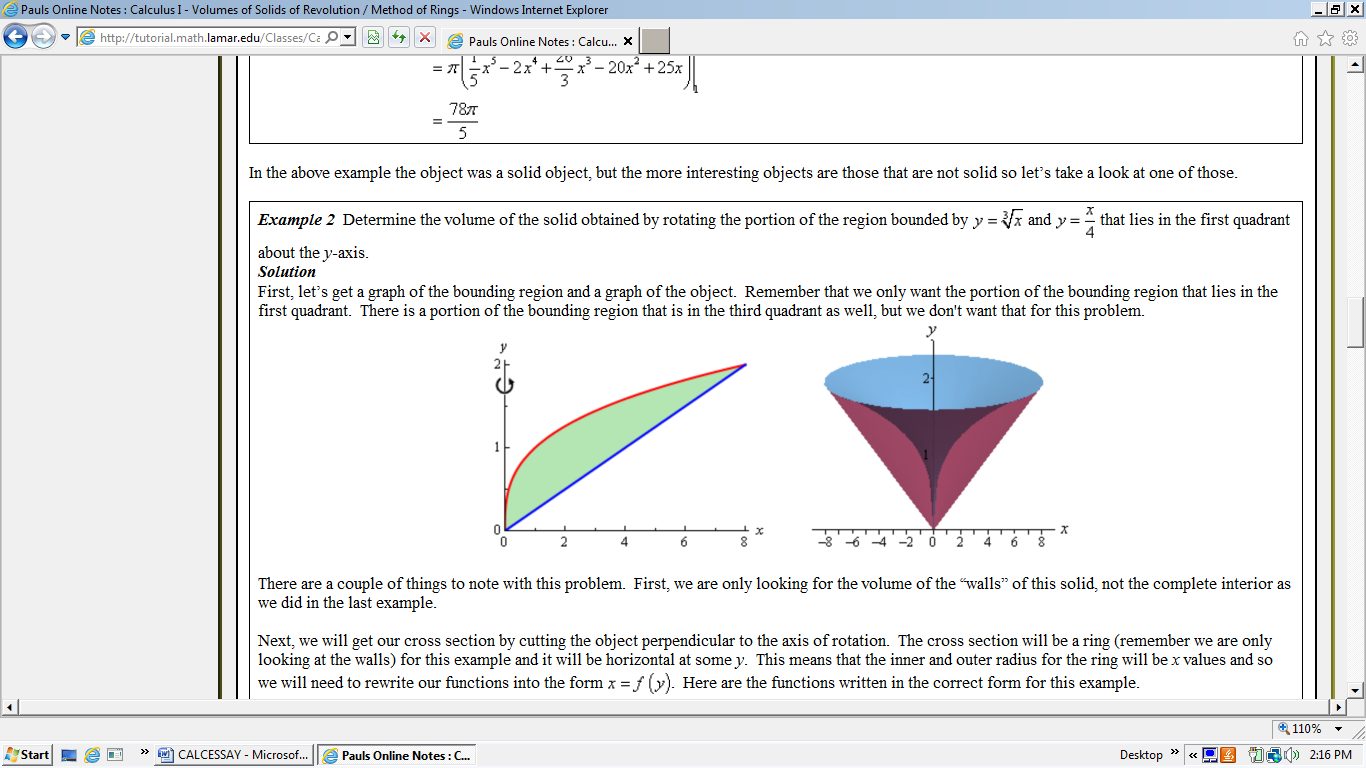
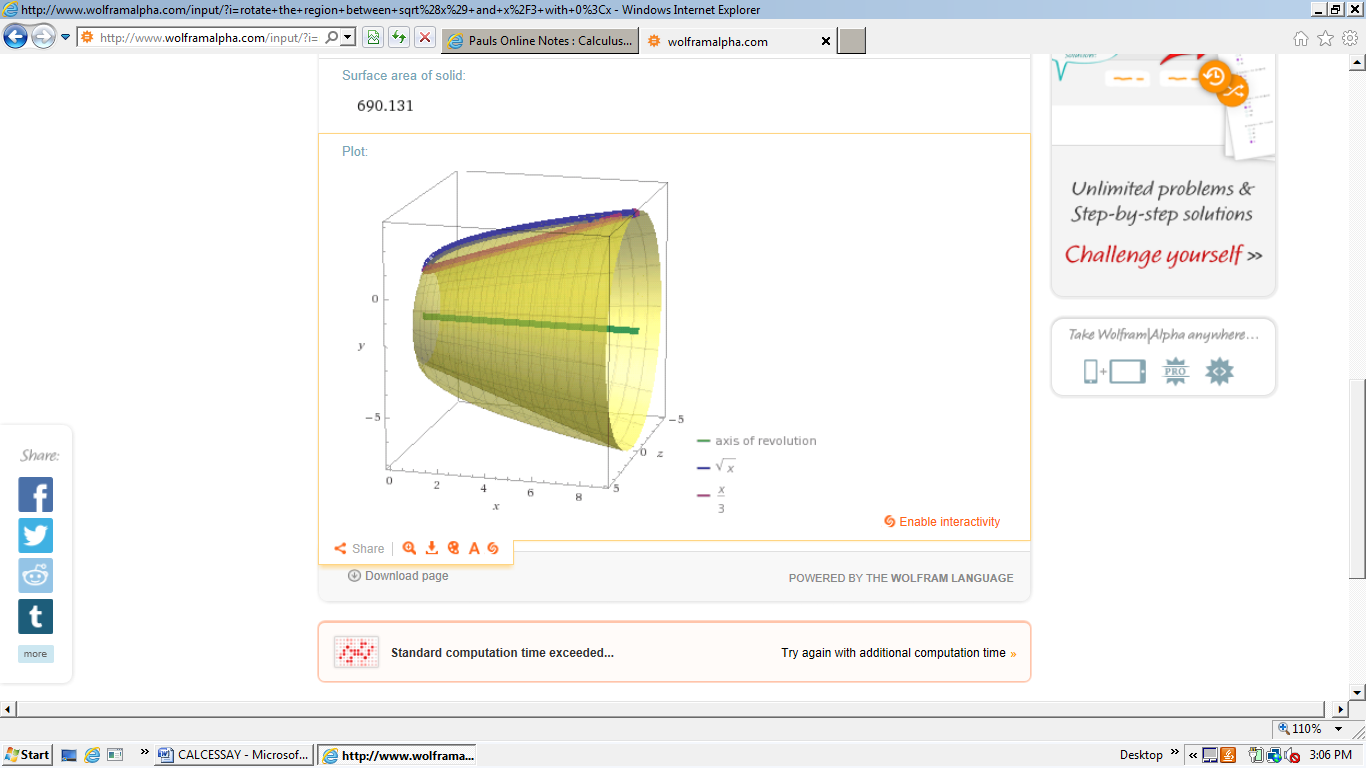


Figure 3. The Ring Method (“Paul’s Online Notes: Rings”)

The green shaded area above in Figure 3 is bound between points 0 and 2 and the two functions. The green shaded area is then rotated around the y-axis. The shaded rotated region is cut into an infinite amount of slices essentially making rings. To find the

volume, the definite integral for rings is utilized. The definite integral is:

In the definite integral above, the points b and a are the endpoints of the rotated ring method. In relation to Figure 3, point b would be 2 and point a would be 0. These points are in terms of the y-coordinates because the functions are rotated around the y-axis. The big radius is the distance from the axis or line to the outermost or upmost function. The small radius is the distance from the axis or line to the innermost function. In relation to Figure 3, the big radius of the graph is the distance from the y-axis to the outermost function itself. The radius in this case would just be the function itself. In relation to Figure 3, the smaller radius of the graph is the distance from the y-axis to the innermost function itself. The radius in this case would just be the function itself. Since the graph is rotated around the y-axis, the graph would be sliced into a dy cut thus utilizing y coordinates as end points. The entire integral is then multiplied by because the area of a ring is and when multiplied by dy, the thickness, you get the volume of the infinite amount of rings added all together which creates the rotated solid.

Figure 4. Applying the Ring Method (*Wolfram Alpha*)

When rotating the sample problem of and around the line of the ring method is the appropriate method because when the graph is rotated around the line, there is a hole in the center, so the disk method would not be appropriate. The method appropriate for this computation of volume would be rings. To apply the ring method, the sample problem of and were graphed in order to find the volume rotated around the line. The definite integral for rings is used to find the volume of the region. A sample set up and solution is displayed below to help with the understanding of the concept.

The end points are 9 and 0 because that is where the two graphs intersect. 2 is added to each radius because the line is 2 units below the x-axis. This change would result in the radius having an extra 2 added to it. The thicknesses of the rings are dx, because the graph is being sliced perpendicular to the x-axis. The volume of the rotated solid in Figure 4 is 31.5 units3.

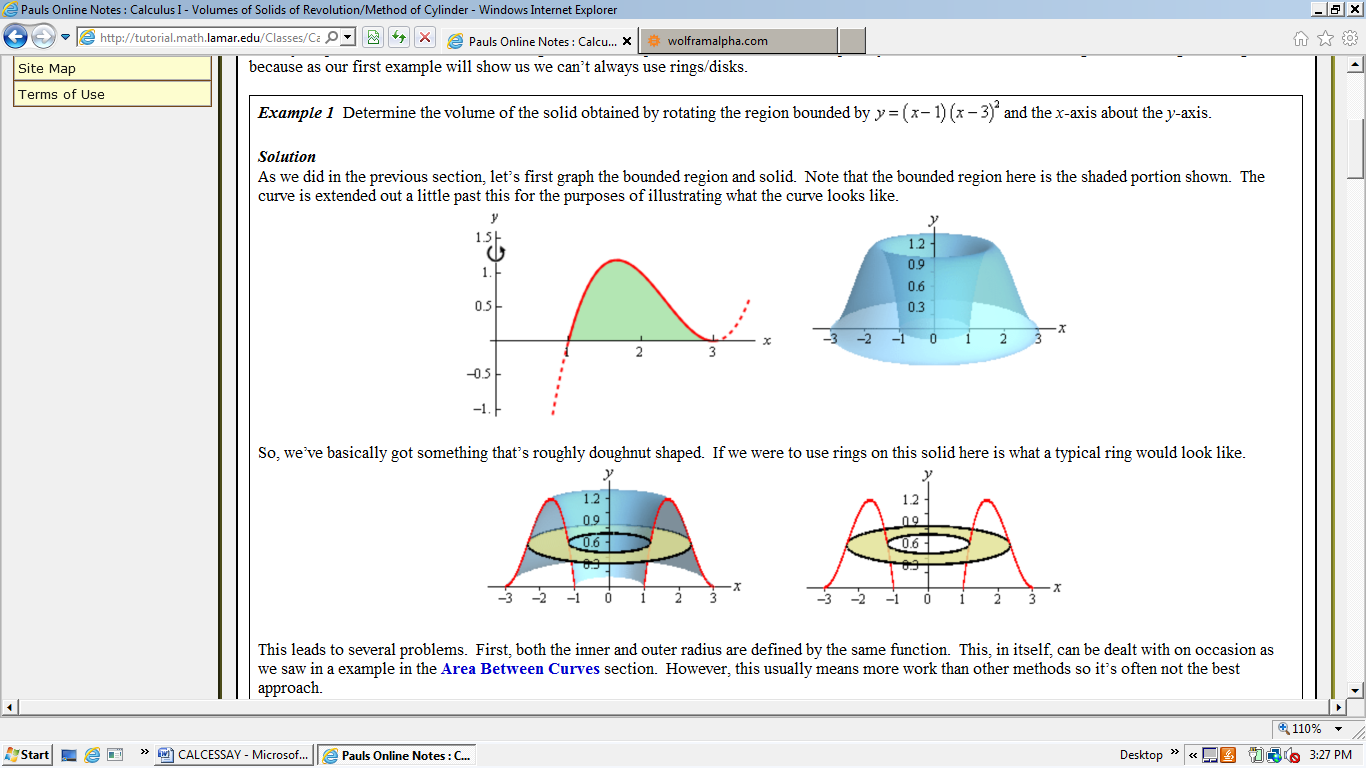
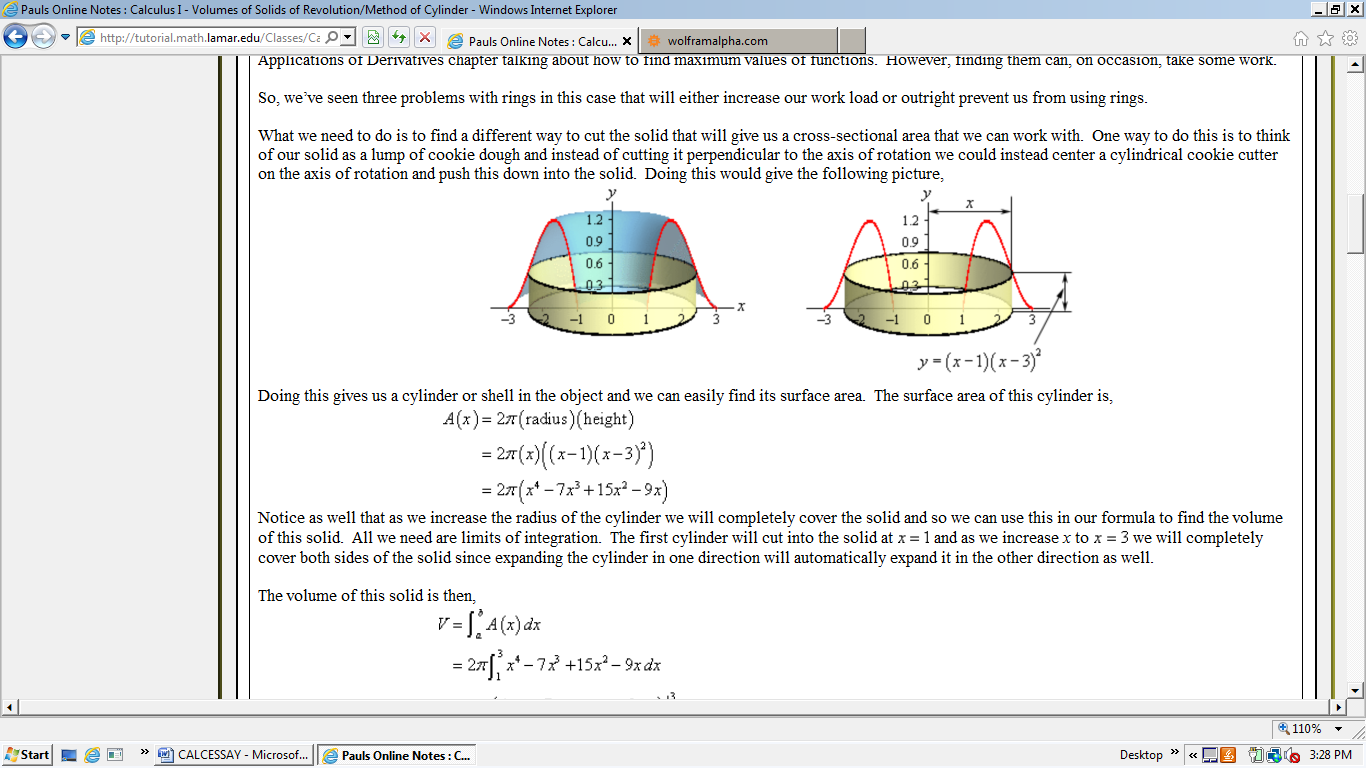
 

Figure 5. The Shell Method (“Paul’s Online Notes: Cylinder”)

The green shaded area above in Figure 5 is bound between points 1 and 3, the function, and the x-axis. The green shaded area is then rotated around the y-axis. The shaded rotated region is cut into an infinite amount of slices essentially making rings. Those rings have heights that are brought down to the x-axis thus making cylindrical shells. To find the volume, the definite integral for shells is utilized. The definite integral is:

In the definite integral above, the points b and a are the endpoints of the rotated shell. In relation to Figure 5, point b would be 3 and point a would be 1. These points are in terms of the x-coordinates even though the function is rotated around the y-axis. This is because the shells are being sliced vertically along with their height which results in a dx cut. The radius is the distance from the axis or line to the outside of the shell. This distance is from the center of the shell to the outer wall. The height is the distance from the bottom of the shell to the top of the shell. In relation to Figure 5 the radius is x, since the values are changing for the radius. In relation to Figure 5, the height would be the function because with each cylindrical shell the height changes based on the function itself. Since the graph is rotated around the y-axis, the graph would be sliced into dx cuts thus utilizing x coordinates as end points. The entire integral is then multiplied by because you are finding the volume of the rotated solid.

The final step in mastering the concept of solids of revolution is understanding the final method for finding volume of a solid. The final method is the cross-section method also known as the slab method. With this method, 3D objects use the area between functions or the area between a function and an axis as the base of the solid. The objects essentially just protrude upwards from the graph.

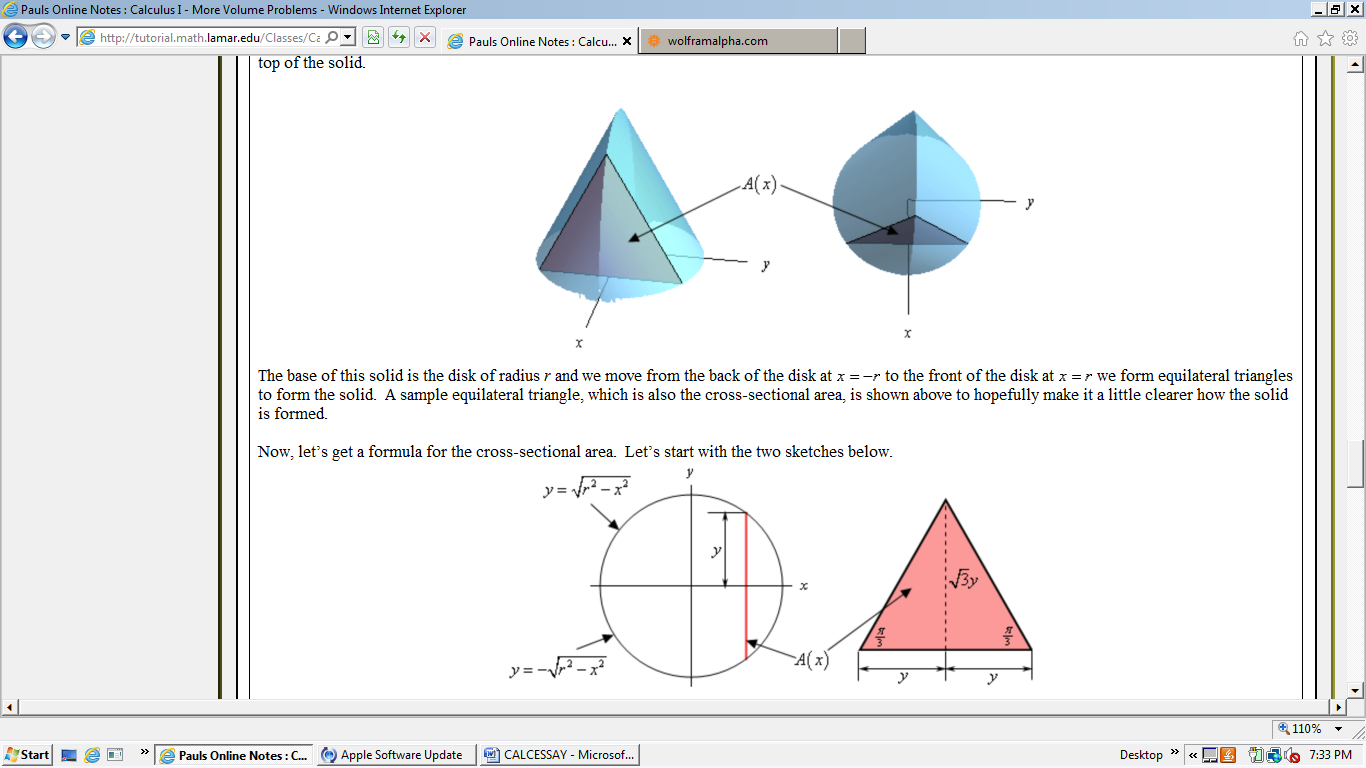
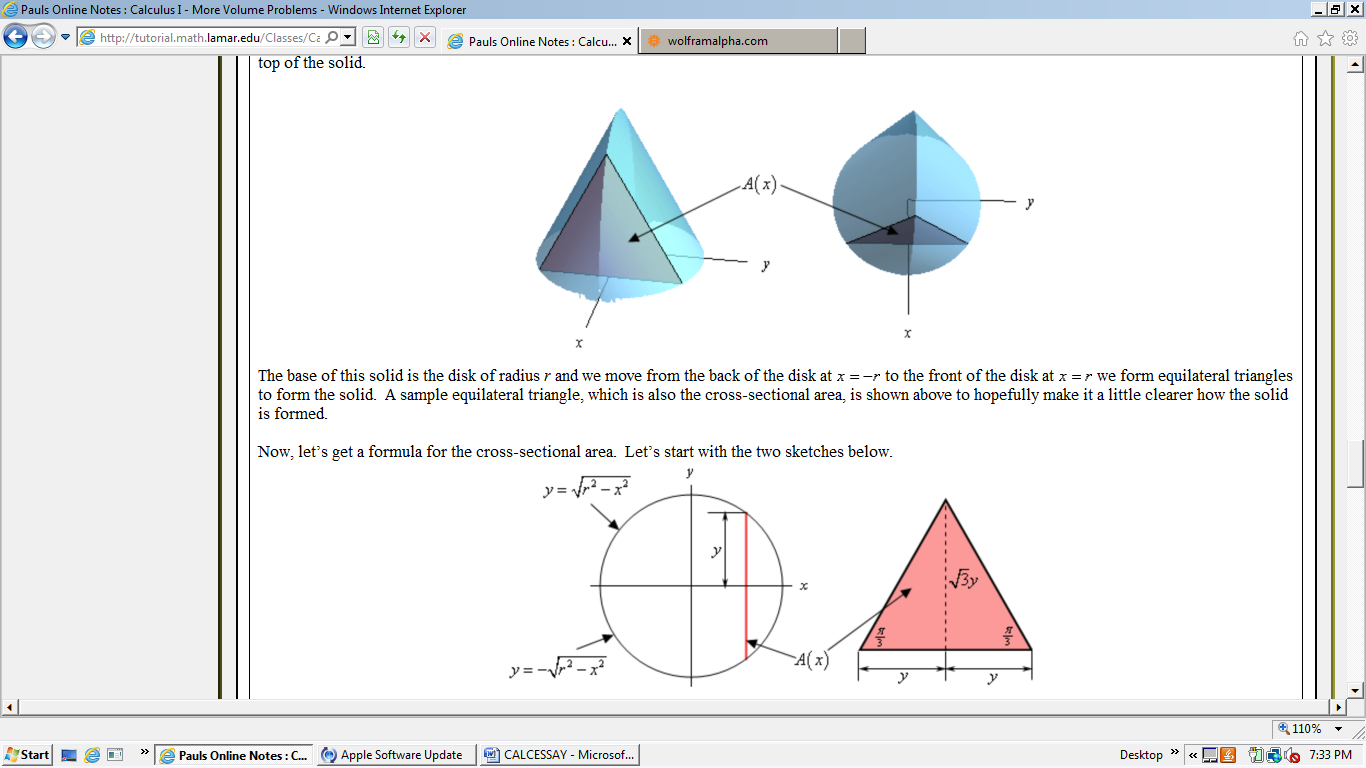
 

Figure 6. Cross-Section (Slab) Method (“Paul’s Online Notes: More Volume”)

The blue shaded area above in figure 6 is bound by the function. The circular function is the base of the solid protruding the graph. The solid is composed of an infinite amount of a given shape that protrudes from the base. To find the volume, the definite integral for slabs is utilized. The definite integral is:

In the definite integral above, the points b and a are the endpoints of function. In relation to Figure 6, these points are in terms of the x-coordinates because the triangles are perpendicular to the x axis. If the triangles were perpendicular to the y axis, then the y-coordinates would be utilized. Since the shapes are being cut along the x-axis, the graph in Figure 6 would follow a dx cut. The area in integral represents the area of the given shape. In relation to Figure 6, the area for the integral would be since it is the area of a triangle. Since the shapes are protruding perpendicular to the x-axis, the graph would be sliced into dx cuts thus utilizing x coordinates as end points.

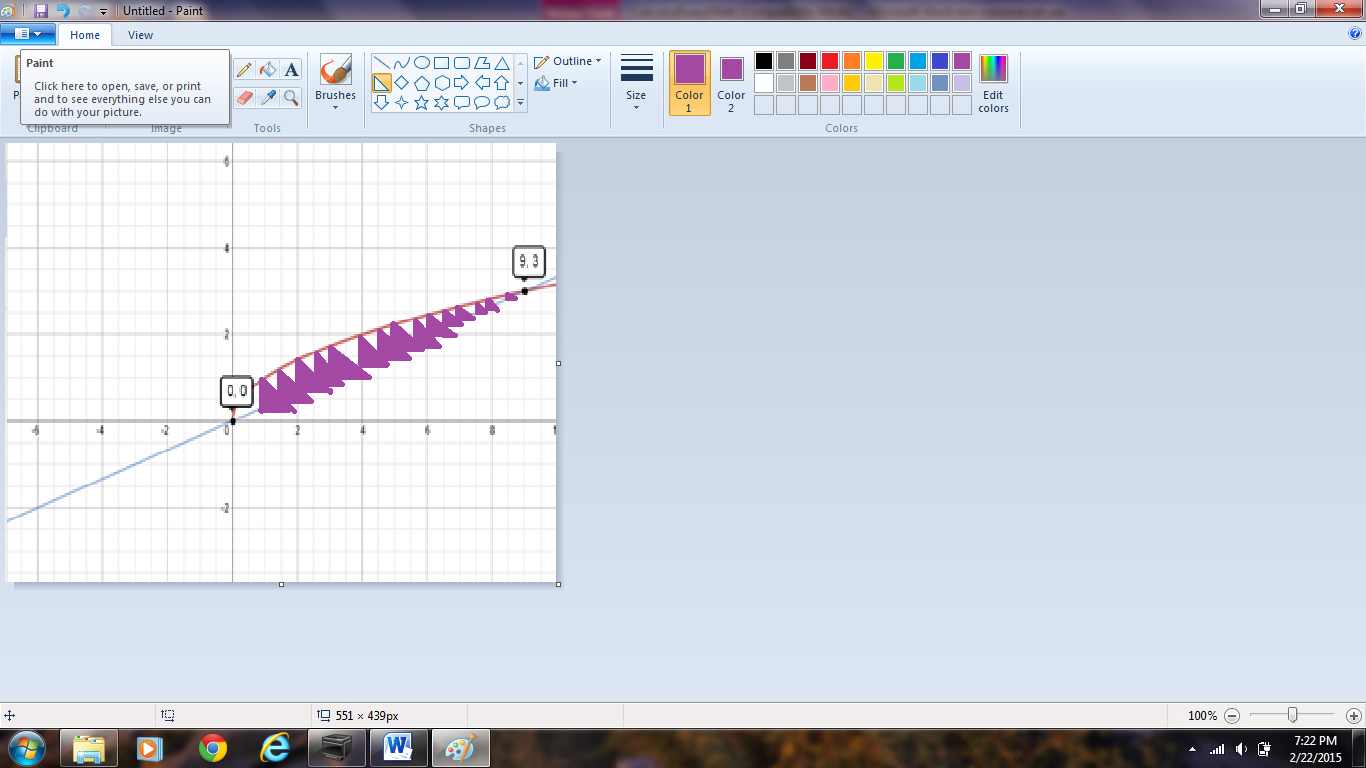


Figure 7. Applying the Cross-Section Method (“Desmos Graphing Calculator”)

Figure 7 above displays the base of the isosceles right triangles bound by and . Only a few purple isosceles right triangles are displayed so it is easy to see how the shapes fit in. In the real life situation there would be an infinite amount of isosceles right triangles extrude the graph in an upward fashion that complete one whole solid. A sample set up and solution is displayed below to help with the understanding of the concept.

The end points of the integral are 9 and 0 because they are the intersection points of the graph. The area of an isosceles right triangle is 1/2side\*side\*sin(included angle). The included angle is 90° because the legs are the same length. The thickness of each triangle is dx because the shapes are perpendicular to the x-axis. The volume of the solid in Figure 7 is 1.35 units3.

In conclusion, solids of revolution is a simple calculus topic that can answer many extraordinary questions. The concept of solids of revolution can be mastered through the several steps explained. As discussed above, the first step to master is finding the area under a curve or the area between two functions. The second step is actually finding the volume of the solid through rotation around an axis or a given line with the shell, disk, or ring methods. The last step is finding the volume of an arbitrary object while utilizing the cross-section method is the final idea that concluded the topic of solids of revolution. Mastering the concept of solids of revolution is the beginning step to understanding the hundreds of other calculus concepts that students are eager to learn.

Works Cited

"Desmos Graphing Calculator." *Desmos Graphing Calculator*. N.p., n.d. Web. 18 Feb. 2015. <https://www.desmos.com/calculator>.

"Pauls Online Notes : Calculus I - More Volume Problems." *Pauls Online Notes : Calculus I - More Volume Problems*. N.p., n.d. Web. 19 Feb. 2015. <http://tutorial.math.lamar.edu/Classes/CalcI/MoreVolume.aspx>.

"Pauls Online Notes : Calculus I - Volumes of Solids of Revolution/Method of Cylinder." *Pauls Online Notes : Calculus I - Volumes of Solids of Revolution/Method of Cylinder*. N.p., n.d. Web. 19 Feb. 2015. <http://tutorial.math.lamar.edu/Classes/CalcI/VolumeWithCylinder.aspx>.

"Pauls Online Notes : Calculus I - Volumes of Solids of Revolution / Method of Rings." *Pauls Online Notes : Calculus I - Volumes of Solids of Revolution / Method of Rings*. N.p., n.d. Web. 19 Feb. 2015. <http://tutorial.math.lamar.edu/Classes/CalcI/VolumeWithRings.aspx>.

*Wolfram Alpha*. N.p., n.d. Web. 19 Feb. 2015. <http%3A%2F%2Fwww.wolframalpha.com%2Finput%2F%3Fi%3Drotate%2Bthe %2Bregion%2Bbetween%2Bsqrt%2528x%2529%2Band%2Bx%252F3%2Bwith %2B0%253Cx%253C9%2Baround%2Bthe%2By%253D-2>.